基本不等式拓展教学实践

226100

2017 2020

$$\geq\sqrt{ab}\geq\frac{2}{\frac{1}{a}+\frac{1}{b}}(a>0,b>0)$$

$$\frac{a+b}{2}\geq\sqrt{ab}(a\geq0,b\geq0)$$

2

2 $CD\leq OD$ O C

$$\sqrt{ab}\leq\frac{a+b}{2}(a\geq0,b\geq0)$$

$a=b$

$$\frac{a^2+b^2}{a+b}\geq\sqrt{\frac{a^2+b^2}{2}}\geq\frac{a+b}{2}$$

$$\geq\sqrt{ab}\geq\frac{2}{\frac{1}{a}+\frac{1}{b}}(a>0,b>0)$$

1

1 45 1 AB

C AB $AC=a$ $BC=b$

C AB DE AD,BD

1 ΔACD

ΔDCB

$CD=\sqrt{ab}$ CD

1 3 C OD CE

RtΔOCD $DC^2=DE\cdot OD$

$$\sqrt{ab}\leq\frac{a+b}{2}$$

C

$a=b$

1 51~52 2 AB $\odot O$

$AC=a$ $CB=b$ C $CD\perp AB$ $\odot O$

D AD,BD ΔACD

ΔDCB $\frac{CD}{CB}=\frac{CA}{CD}$ $CD=\sqrt{ab}$ $OD=\frac{a+b}{2}$

$CD\leq OD$ $\sqrt{ab}\leq\frac{a+b}{2}$ C O

$a=b$

2

$$\frac{a^2+b^2}{a+b}\geq\sqrt{\frac{a^2+b^2}{2}}\geq\frac{a+b}{2}$$

$\sqrt{\frac{a^2+b^2}{2}}$

2

$\frac{a-b}{2}$ $\sqrt{\frac{a^2+b^2}{2}}$

3 2 $\frac{a-b}{2}$

1 OC 4 $OF \perp AB$

CF $CF=\sqrt{\frac{a^2+b^2}{2}}$

4

3

CF

4

4 5 C CF OF

G FG $\frac{a^2+b^2}{a+b}$

5

4

$\frac{a^2+b^2}{a+b}$

4

$\frac{a^2+b^2}{a+b} \geq \sqrt{\frac{a^2+b^2}{2}} \geq \frac{a+b}{2}$

$\geq \sqrt{ab} \geq \frac{2}{\frac{1}{a}+\frac{1}{b}} (a>0, b>0)$

1 70 16 6 $ABDC$

$AB=a$ $CD=b$ O

GH

KL

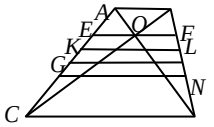
$ABLK$ $KLDC$ EF

O MN

$ABDC$

GH, KL, EF, MN

$\frac{a+b}{2}, \sqrt{ab}, \frac{2}{\frac{1}{a}+\frac{1}{b}}, \sqrt{\frac{a^2+b^2}{2}}$



6

5 $GH=\frac{a+b}{2}$ $KL=\sqrt{ab}$ $EF=\frac{2}{\frac{1}{a}+\frac{1}{b}}$

$MN=\sqrt{\frac{a^2+b^2}{2}}$ $MN>GH>KL>EF$

$\sqrt{\frac{a^2+b^2}{2}}>\frac{a+b}{2}>\sqrt{ab}>\frac{2}{\frac{1}{a}+\frac{1}{b}}$

6 $a=b$

5 6

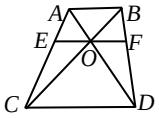
$EF=\frac{2}{\frac{1}{a}+\frac{1}{b}}$ $MN=\sqrt{\frac{a^2+b^2}{2}}$

7 $ABDC$ O O

AC, BD E, F

$AB=a$ $CD=b$ 1 $OE=OF$ 2

$EF=\frac{2}{\frac{1}{a}+\frac{1}{b}}$



7

4

$OE=\frac{1}{\frac{1}{a}+\frac{1}{b}}$

$\begin{cases} \frac{OE}{b}=\frac{AO}{AD}, \\ \frac{OE}{a}=\frac{OC}{BC}. \end{cases}$

$\frac{AO}{AD}+\frac{OC}{BC}=\frac{AO}{AD}+\frac{OD}{AD}=1$

$OE = \dfrac{1}{\dfrac{1}{a}+\dfrac{1}{b}}$ 4

EF

$MN = \sqrt{\dfrac{a^2+b^2}{2}}$

4

4

$\dfrac{a^2+b^2}{a+b}$

h_1, h_2

7 $MN = t$

4

$\dfrac{a+t}{2}\cdot h_1 = \dfrac{b+t}{2}\cdot h_2 = \dfrac{a+b}{4}\cdot(h_1+h_2)$ h_1, h_2

t

EF $EF = \dfrac{2}{\dfrac{1}{a}+\dfrac{1}{b}}$ 7

5

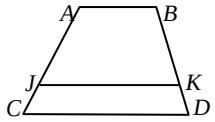
10

$\dfrac{AE}{EC} = \dfrac{a}{b}$

$\dfrac{a+t}{2}\cdot h_1 = \dfrac{b+t}{2}\cdot h_2 = \dfrac{a+b}{4}\cdot(h_1+h_2)$

$\dfrac{AJ}{JC} = \dfrac{b}{a}$ 8 JK

$$\begin{cases} \dfrac{h_1}{h_1+h_2} = \dfrac{a+b}{2(a+t)}, \\ \dfrac{h_2}{h_1+h_2} = \dfrac{a+b}{2(b+t)}, \end{cases}$$



8

$$2 = \frac{a+b}{a+t}+\frac{a+b}{b+t}$$

JK

$$2 = \frac{a+t+(b-t)}{a+t}+\frac{b+t+(a-t)}{b+t}$$

JK

$0 = \dfrac{b-t}{a+t}+\dfrac{a-t}{b+t}$ $0 = b^2-t^2+a^2-t^2$

3

$t^2 = \dfrac{a^2+b^2}{2}$ $t = \sqrt{\dfrac{a^2+b^2}{2}}$

8 h_1, h_2

$$\frac{h_1}{h_1+h_2}+\frac{h_2}{h_1+h_2} = 1$$

$$2 = \frac{a+b}{a+t}+\frac{a+b}{b+t}$$

$\geq$ $\geq$

$\geq$ $\geq$

MN